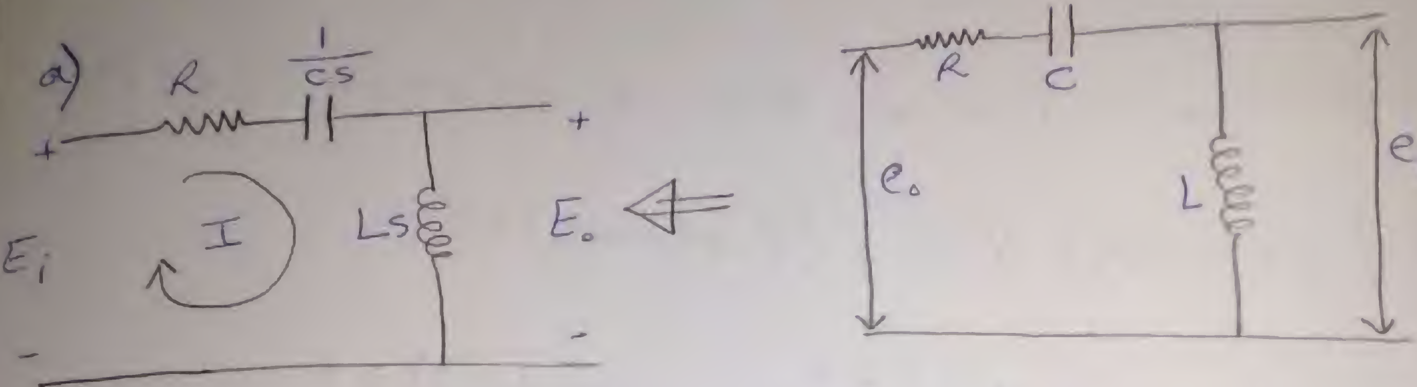
Sheet 2

1) Drive the transfer function $\frac{E_o}{E_i}$ for the RLC circuits shown in figure below:-



<u>Sol</u>

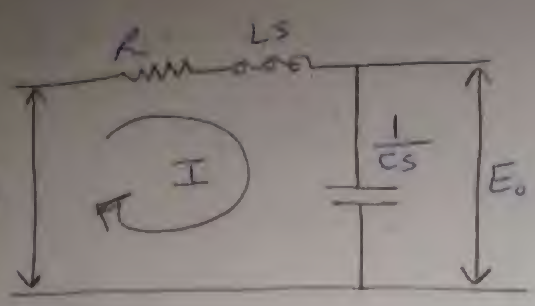
$$E_i(s) = \left(R + Ls + \frac{1}{Cs}\right) I \quad \& \quad E_o(s) = I L s$$

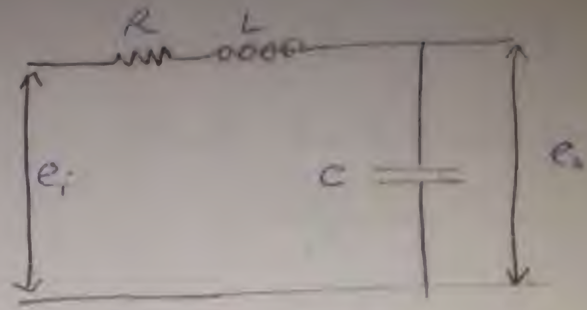
$$E_i(s) = \left(R + Ls + \frac{1}{Cs}\right) \frac{E_o}{Ls}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Ls}{R + Ls + \frac{1}{Cs}}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{CLs^2}{1 + RCs + LCs^2}}$$

b)



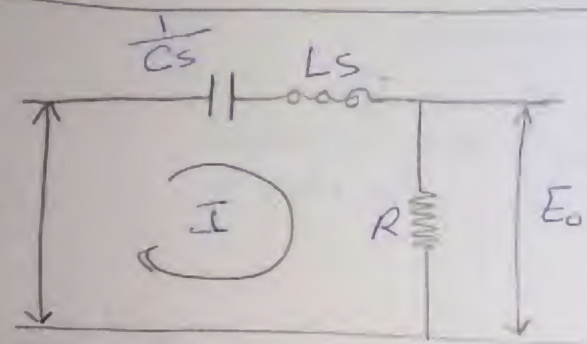


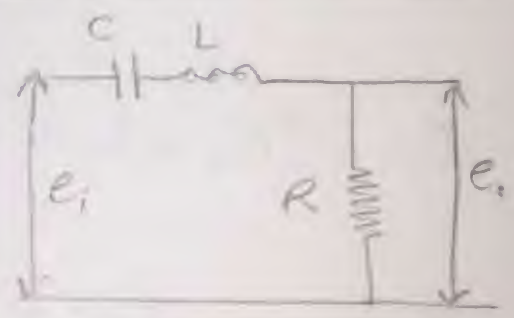
$$E_i(s) = \left(R + LS + \frac{1}{Cs}\right) I \quad ; \quad E_o(s) = \frac{I}{Cs}$$

$$\therefore E_i(s) = \left(R + LS + \frac{1}{Cs}\right) E_o(s) * Cs$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{RCS + CLS^2 + 1}}$$

c)





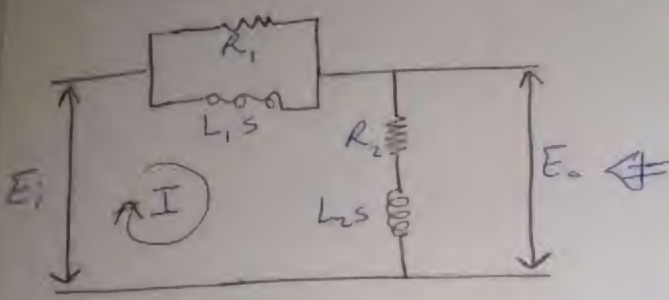
$$E_i(s) = \left(R + LS + \frac{1}{Cs}\right) I \quad ; \quad E_o(s) = IR$$

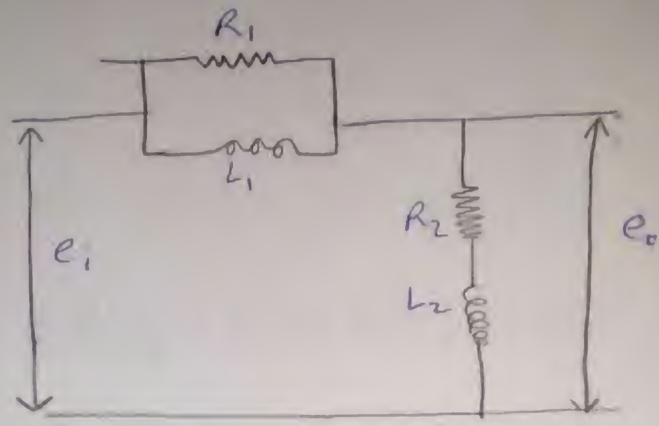
$$E_i(s) = \left(R + LS + \frac{1}{Cs}\right) * \frac{E_o(s)}{R}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R}{R + LS + \frac{1}{Cs}} = \boxed{\frac{RCS}{RCS + LCS^2 + 1}}$$

[2] For electrical networks shown in figure below assuming zero initial conditions, obtain the transfer function E_o/E_i.

[a]





$$E_i = \left(R_2 + L_2 s + \frac{R_1 L_1 s}{R_1 + L_1 s}\right) I$$

$$E_o = I\,(R_2 + L_2 s)$$

$$E_i = \left(R_2 + L_2 s + \frac{R_1 L_1 s}{R_1 + L_1 s}\right) * \frac{E_o}{R_2 + L_2 s}$$

$$\frac{E_o}{E_i} = \frac{R_2 + L_2 s}{R_2 + L_2 s + \dfrac{R_1 L_1 s}{R_1 + L_1 s}} \qquad * (R_1 + L_1 s)$$

$$\frac{E_o}{E_i} = \frac{(R_1 + L_1 s)(R_2 + L_2 s)}{R_1 R_2 + R_2 L_1 s + R_1 L_2 s + L_1 L_2 s^2 + R_1 L_1 s}$$

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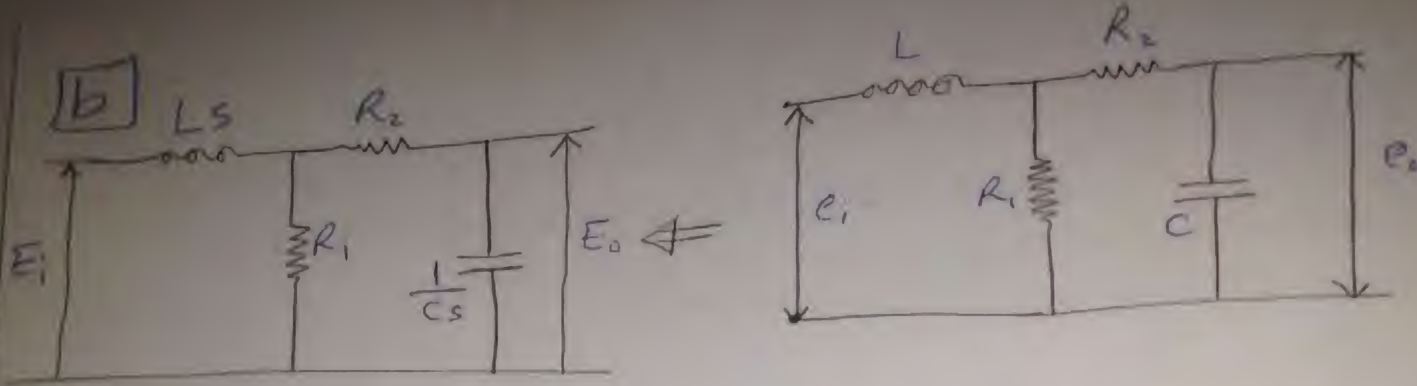
$\boxed{b}$



$$E_i(s) = I\,Ls + V_o \qquad ; V_o = I\,X$$

$$X = R_1 \,//\, \left(R_2 + \frac{1}{Cs}\right)$$

$$E_i(s) = \left[Ls + \left(R_1 \,//\, \left(R_2 + \frac{1}{Cs}\right)\right)\right] * I$$

$$E_o(s) = \frac{I}{Cs}$$

$$E_i(s) = \left[Ls + R_1 \,//\, \left(R_2 + \frac{1}{Cs}\right)\right] * E_o(s) * Cs$$

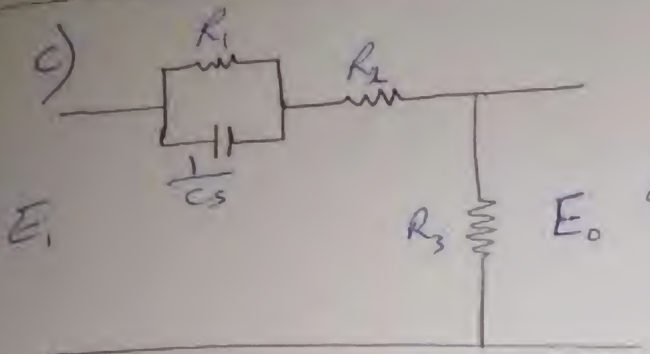
$$= \left[Ls + \frac{R_1\left(R_2 + \frac{1}{Cs}\right)}{R_1 + R_2 + \frac{1}{Cs}}\right] * E_o(s) * Cs$$

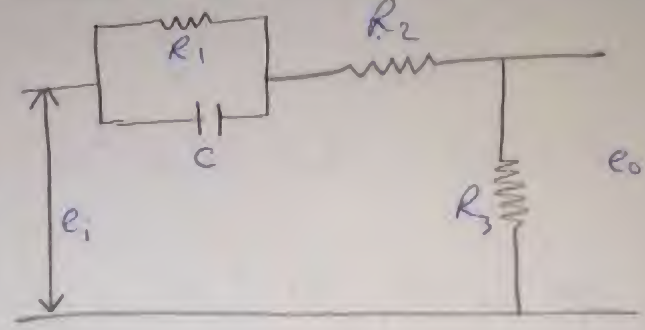
$$= \left[LCs^2 + \frac{R_1R_2Cs + R_1}{R_1 + R_2 + \frac{1}{Cs}}\right] E_o(s)$$

$$E_i(s) = \left[\frac{R_1LCs^2 + R_2LCs^2 + Ls + R_1R_2Cs + R_1}{R_1 + R_2 + \frac{1}{Cs}}\right] E_o(s)$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_1 + R_2 + \frac{1}{Cs}}{R_1 LCs^2 + R_2 LCs^2 + Ls + R_1 R_2 Cs + R_1}}$$

c)





$$E_i(s) = \left[R_2 + R_3 + \left(R_1 \,//\, \frac{1}{Cs}\right)\right] I$$

$$= \left[R_2 + R_3 + \frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}}\right] I$$

$$= \left[R_2 + R_3 + \frac{R_1}{1 + R_1 Cs}\right] I \quad \cancel{= \frac{R_2 + R_1 R_2 Cs + R_3 + R_1 R_3 Cs +}{1 + R_1 Cs}}$$

$$= \left[\frac{R_2 + R_1 R_2 Cs + R_3 + R_1 R_3 Cs + R_1 + R_1^2 Cs}{1 + R_1 Cs}\right] I$$

$$E_o(s) = I R_3$$

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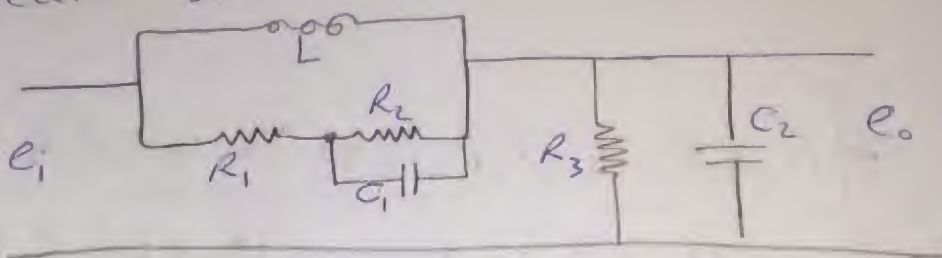
$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_3 + R_1 R_3 C s}{R_2 + R_1 R_2 C s + R_3 + R_1 R_3 C s + R_1 + R_1^2 C s}}$$

3 using Crammer rule, find the transfer function $\frac{E_o}{E_i}$ for the circuit given below.



$$E_i = \left(Z + \frac{R_3 * \frac{1}{C_2 s}}{R_3 + \frac{1}{C_2 s}} \right) I$$

$$\frac{E_i}{I} = Z + \frac{R_3}{1 + R_3 C_2 s}$$

$$Z_1 = \frac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} + R_1 = \frac{R_2}{1 + R_2 C_2 s} + R_1 = \frac{R_2 + R_1 + R_1 R_2 C_2 s}{1 + R_2 C_2 s}$$

$$Z = Z_1 // Ls = \frac{\frac{R_2 + R_1 + R_1 R_2 C_2 s}{1 + R_2 C_2 s} * Ls}{\frac{R_2 + R_1 + R_1 R_2 C_2 s}{1 + R_2 C_2 s} + Ls} =$$

$$Z = \frac{(R_1+R_2)Ls + R_1R_2LC_1s^2}{(R_1+R_2) + R_1R_2C_1s + Ls + LR_2C_1s^2}$$

$$\frac{E_i}{I} = \frac{L(R_1+R_2)s + R_1R_2LC_1s^2}{(R_1+R_2)+R_1R_2C_1s+Ls+LR_2C_1s^2} + \frac{R_3}{1+R_3C_2s}$$

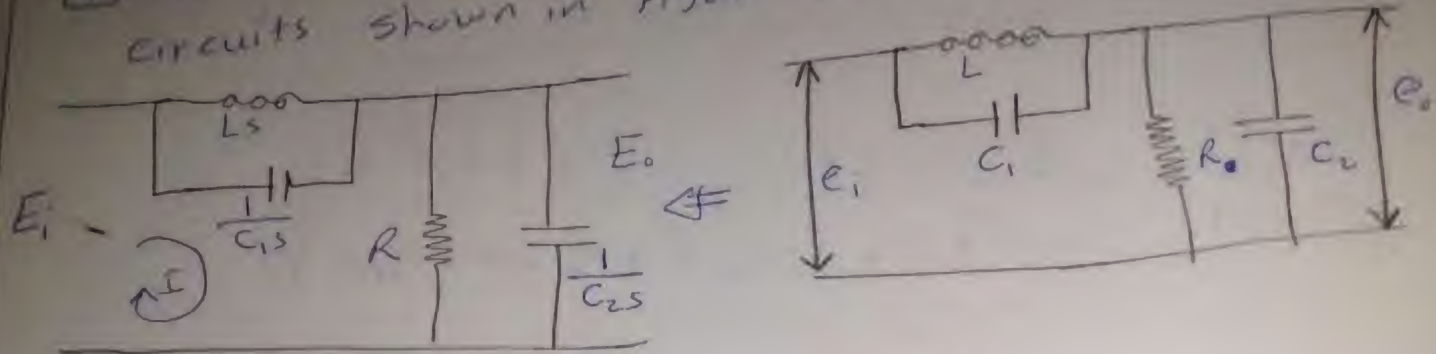
$$\frac{I}{E_i} = \frac{\left[R_1+R_2 + (R_1R_2C_1+L)s + LR_2C_1s^2\right](1+R_3C_2s)}{(R_1+R_2)R_3 + R_3(R_1R_2C_1+L)s + LR_2R_3C_1s^2 + (1+R_3C_2s) * \left[L(R_1+R_2)s + R_1R_2LC_1s^2\right]}$$

$\rightarrow *$

$$\frac{E_o}{I} = \frac{1}{C_2s}$$

$$\frac{E_o}{E_i} = * \quad * \frac{1}{C_2s}$$

[9] obtain the transfer function E_o/E_i for the circuits shown in figure below..



$$E_i(s) = \left(\left[Ls \,//\, \frac{1}{C_1 s}\right] + \left[R \,//\, \frac{1}{C_2 s}\right]\right) * I$$

$$= \left[\frac{\frac{L}{C_1}}{Ls + \frac{1}{C_1 s}} + \frac{\frac{R}{C_2 s}}{R + \frac{1}{C_2 s}}\right] * I$$

$$= \left(\frac{Ls}{1 + LC_1 s^2} + \frac{R}{RC_2 s + 1}\right) I$$

$$= \frac{Ls + RLC_2 s^2 + R + RLC_1 s^2}{(1 + LC_1 s^2)(RC_2 s + 1)} * I$$

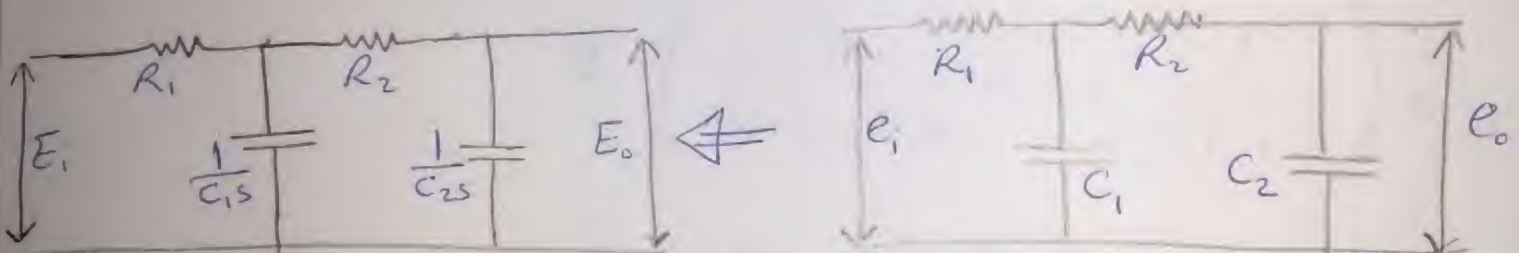
$$E_i(s) = \frac{Ls + RLC_2 s^2 + R + RLC_1 s^2}{RC_2 s + 1 + RLC_1 C_2 s^3 + LC_1 s^2} * I$$

$$E_o(s) = \frac{I}{C_2 s}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{RC_2 s + RLC_1C_2 s^3 + LC_1 s^2 + 1}{C_2 s\left(Ls + RLC_2 s^2 + R + RLC_1 s^2\right)}$$

$$\frac{E_o}{E_i} = \frac{RC_2 s + RLC_1 C_2 s^3 + LC_1 s^2 + 1}{C_2 L s^2 + RLC_2^2 s^3 + R + RLC_1 C_2 s^3}$$

$\boxed{5}$ Drive the transfer function E_o/E_i for the RC ladder network given in figure below:-



$$E_i(s) = IR_1 + V_o \quad ; \quad V_o = IX$$

$$X = \frac{1}{C_1 s} \,//\, \left(R_2 + \frac{1}{C_2 s}\right)$$

$$= \frac{\frac{1}{C_1 s}\left(R_2 + \frac{1}{C_2 s}\right)}{\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s}}$$

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$$X = \frac{\frac{R_2}{C_1 s} + \frac{1}{C_1 C_2 s^2}}{\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s}} \quad * C_1 C_2 s^2$$

$$X = \frac{R_2 C_2 s + 1}{C_2 s + R_2 C_1 C_2 s^2 + C_1 s}$$

$$E_i(s) = \left[R_1 + \frac{R_2 C_2 s + 1}{C_2 s + R_2 C_1 C_2 s^2 + C_1 s} \right] * I$$

$$= \left[\frac{R_1 C_2 s + R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_2 C_2 s + 1}{C_2 s + R_2 C_1 C_2 s^2 + C_1 s} \right] * I$$

$$E_o(s) = \frac{I_2}{C_2 s} \quad ; \quad I_2 = I \frac{\frac{1}{C_1 s}}{R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$I_2 = \frac{I}{R_2 C_1 s + 1 + \frac{C_1 s}{C_2 s}} = \frac{I C_2 s}{R_2 C_1 C_2 s^2 + C_2 s + C_1 s}$$

$$E_o(s) = \frac{I}{R_2 C_1 C_2 s^2 + C_2 s + C_1 s}$$

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$$\frac{E_o(s)}{E_i(s)} = \frac{C_1 s + C_2 s + R_2 C_1 C_2 s^2}{(R_2 C_1 C_2 s^2 + C_2 s + C_1 s)(R_1 C_2 s + R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_2 C_2 s + 1)}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{R_1 C_2 s + R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_2 C_2 s + 1}}$$

$\boxed{11}$ Sheet 2